Lec (06)

Time Varying Theory

Study "2" theories For Maxwell' equation But in Magnetic / Electric Time Varying.

$\boxed{1^{st}\text{ equation}}$ • iF the Magnetic Flux lines which induces a wire loop is varying with time, ∴ The Induced Current will Results & due to this Current:- a potential (voltage) is Induced ∴

$$\boxed{V = -\frac{\partial \Phi}{\partial t}}$$

Induced voltage ∴ V; Φ → Magnetic flux

• EMF about closed Path is equal to Line Integral of Electric field about the path

$$\boxed{Emf = \oint_c \bar{E} \cdot \overline{dl}}$$

→ electromotive force

القوة الدافعة الكهربية بتعتمد على تحرك الـ E field

$emf = \oint_c E \cdot dl$

• Faraday's says that $\oint \bar{E} \cdot \overline{dl}$ = Negative Rate of change of Magnetic flux density on surface

لو عندي مسار مغلق ∴ مجموع E على الـ line = مجموع معدل تغير B بالنسبة للزمن على الـ surface

$$V_{emf} = \boxed{\oint \bar{E} \cdot \overline{dl} = \iint_s -\frac{\partial B}{\partial t} \cdot \overline{ds}}$$

• Now From Stock theorem

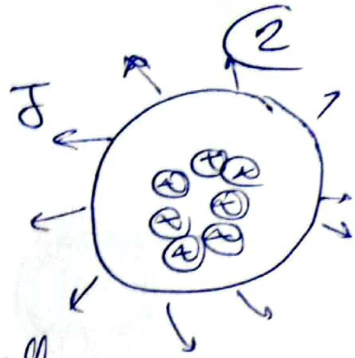
نحول التكامل الـ line للتكامل السطحي

$$\therefore \oint \bar{E} \cdot \overline{dl} = \iint_s (\nabla \times \bar{E}) \cdot ds = \iint_s -\frac{\partial \bar{B}}{\partial t} \cdot \overline{ds}$$

$$\therefore \boxed{\nabla \times \bar{E} = -\frac{\partial B}{\partial t}}$$

معادلة ماكسويل الأولى في Time varying

2nd equation



As shown; if a current flow in a Region (Volume), This current will generate current density on the surface of charge within the Region itself & outward current through enclosing surface can be equal to $\oiint \bar{J} \cdot d\bar{s}$

كلما كانت هناك شحنات داخل منطقة مقفوله فإن الشحنات عندما تتحرك تعمل تيار I والتيار لو اقترب من السطح يعمل J ، وكلما قلت الشحنات جوه السطح زادت J هنا & السطح.

if Q_m = Total charge enclosed By Region (volume)

$$\therefore \quad Q_m = \iiint \rho_v \, dv$$

مجموع J على السطح $= I$ ، كلما نقص معدل تغير الشحنة جوه بـ value كلما زاد التيار على السطح

$$\therefore \quad \boxed{\oiint \bar{J} \cdot d\bar{s} = -\frac{\partial}{\partial t} \iiint \rho_v \, dv} \quad [1]$$

معدل نقص الشحنة

• from divergence Theorem

div

تحويل سطح لحجم

$$\boxed{\oiint \bar{J} \cdot d\bar{s} = \iiint \nabla \cdot \bar{J} \, dv} \quad [2]$$

From 1 & 2

$$\therefore \quad \nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \longrightarrow \text{Continuity equation}$$

مهم

• From Ampere law

$$\therefore \quad \nabla \times \bar{H} = \bar{J}$$

لو اخدت div للطرفين

$$\nabla \cdot \nabla \times \bar{H} = \text{zero}$$

div

$$\text{But } \nabla \cdot \bar{J} \neq$$

$$\therefore \quad \nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t}$$

$$\therefore \quad \nabla \cdot \bar{J} + \frac{\partial \rho_v}{\partial t} = 0$$

$$\therefore \quad \nabla \cdot \nabla \times \bar{H} = \left(\nabla \cdot \bar{J} + \frac{\partial \rho_v}{\partial t}\right) = 0$$

نعدلها
ماكسويل حلها

$$\nabla \cdot \nabla \times \overline{H} = \nabla \cdot \overline{J} + \frac{\partial \rho_v}{\partial t} = \nabla \cdot \overline{J} + \frac{\partial \nabla \cdot D}{\partial t}$$

$$\because \boxed{\rho_v = \nabla \cdot D}$$ معادلة ماكس في ال static

$$\therefore \cancel{\nabla} \cdot \nabla \times \overline{H} = \cancel{\nabla} \cdot \overline{J} + \frac{\partial \nabla \cdot D}{\partial t} = \cancel{\nabla} \cdot \frac{\partial \overline{D}}{\partial t} + \cancel{\nabla} \cdot \overline{J}$$

$$\therefore \boxed{\nabla \times \overline{H} = \overline{J_c} + \frac{\partial \overline{D}}{\partial t}}$$ معادلة ماكس رقم 2 في Time varying

types of Current densities J_{cond} & J_d

$$\nabla \times \overline{H} = \underbrace{\overline{J_c}}_{\text{conduction current density} = J_{cond.}} + \overline{J_d}$$

J_d → dielectric current density $= J_d$

J_c → conduction current density $= J_{cond.}$

* Conduction current density (in conductors) $\boxed{J_{con} = \sigma E \;\&\; I_c = \sigma E A}$
* Dielectric (displacement current density) (in capacitors)

$$\boxed{J_d = \frac{\partial \overline{D}}{\partial t} = \epsilon \frac{\partial \overline{E}}{\partial t}}$$

تلخيص معادلات ماكسويل

Static Field (Diff form) diff	Integral	time varying field diff	Int
① $\nabla \times E = 0$	$\oint \overline{E} \cdot \overline{dl} = 0$	$\nabla \times E = -\frac{\partial \overline{B}}{\partial t}$	$\oint E \cdot \overline{dl} = -\frac{\partial}{\partial t} \oint B \cdot ds$
② $\nabla \times \overline{H} = \overline{J}$	$\oint_c \overline{H} \cdot \overline{dl} = \iint J \cdot ds$	$\nabla \times \overline{H} = \overline{J_c} + \frac{\partial \overline{D}}{\partial t}$	$\oint \overline{H} \cdot \overline{dl} = \iint (J + \frac{\partial D}{\partial t}) \cdot ds$
③ $\nabla \cdot \overline{B} = 0$	$\oint \overline{B} \cdot \overline{ds} = 0$	$\nabla \cdot \overline{B} = 0$	$\oint B \cdot ds = 0$
④ $\nabla \cdot \overline{D} = \rho_v$	$\oint \overline{D} \cdot \overline{ds} = \iiint \rho_v \, dv$	$\nabla \cdot \overline{D} = \rho_v$	$\oint \overline{D} \cdot \overline{ds} = \iiint \rho_v \, dv$

From gauss

$$\oint \overline{B} \cdot ds = 0$$

By divergence

$$\oint \overline{B} \cdot \overline{ds} = \iiint \nabla \cdot \overline{B} \, dv = 0$$

$$\therefore \nabla \cdot \overline{B} = 0$$

إثبات

From gauss

$$\oint \overline{D} \cdot \overline{ds} = \iiint \rho_v \, dv$$

By divergence

$$\oint \overline{D} \cdot \overline{ds} = \iiint \nabla \cdot \overline{D} \, dv = \iiint \rho_v \, dv$$

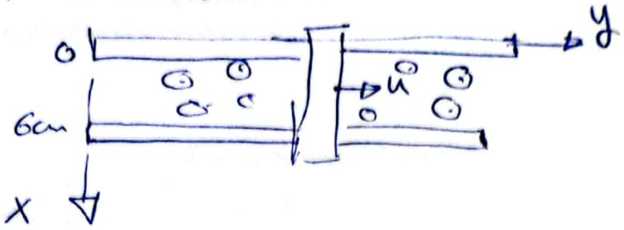
$$\therefore \nabla \cdot \overline{D} = \rho_v$$

إثبات

Ex (01)

A Conductor Bar Can slide freely over 2 Conducting Rails as show in figure. Calc. Induced voltage emf in the Bar If The Bar is stationed at $y=8$ cm &

$$\bar{B} = 4\cos(10^6 t)\,\hat{a}_z \ \ mw/m^2$$



Sol

$$V_{emf} = -\iint \frac{\partial \bar{B}}{\partial t}\cdot \bar{ds}$$

$$\bar{ds} = dx\,dy$$

$$\therefore V_{emf} = -\int_{y=0}^{0.08}\int_{x=0}^{0.06} (-4\times10^6\times10^{-3})\sin(10^6 t)\,dx\,dy$$

$$= +4\times10^3\,[x]_0^{0.06}\,[y]_0^{0.08}\sin 10^6 t$$

$$= 19.2\sin 10^6 t \ \ V$$

Ex (02)

If the Conduction Current Flow through a wire with Conductivity $\sigma = 3\times10^7$ S/m & relative permittivity $\varepsilon_r = 1$ is given By $I_c = 3\sin\omega_c t$ (mA), If $\omega = 10^8$ rad/s find displacement current density J_D.

Sol

$$J_d = \frac{\partial D}{\partial t} = \epsilon\frac{\partial E}{\partial t} \Rightarrow \qquad \longrightarrow D=\epsilon E$$

$$J_c = \sigma E \qquad \therefore I_c = \sigma E A$$

$$\therefore E = \frac{I_c}{\sigma A} = \frac{3\times10^{-3}\sin\omega_c t}{3\times10^7\,A}, \quad \frac{\partial E}{\partial t} = \frac{10^{-10}\omega\cos\omega t}{A}$$

$$J_d = \epsilon * \frac{\omega * 10^{-10}\cos\omega t}{A} = 8.85\times10^{-12}\times1\times10^8\times\frac{10^{-10}}{A}\cos\omega t = \frac{8.85\times10^{-14}}{A}\cos\omega t$$

$$\boxed{I_d = J_d \times A = 8.85\times10^{-14}\cos\omega t}$$

Solved Examples (03)

A Poor Conductor characterized By Conductivity $\sigma = 100$ S/m & $\epsilon_r = 4$, what is angular freq (ω) at which amplitude of conduction current density = Amplitude of displacement current density = $|J_d|$

J_d

sol

$$J_d = \frac{\partial D}{\partial t} = \epsilon \frac{\partial \bar{E}}{\partial t}$$

$$\frac{\partial}{\partial t} = j\omega$$

$$\left|\frac{\partial}{\partial t}\right| = \omega$$

$$\therefore J_d = \epsilon \omega \bar{E}$$

$$|J_d| = \epsilon \omega E$$

$$|J_c| = \sigma E$$

$$\therefore |J_d| = |J_c| \Rightarrow \sigma \cancel{E} = \epsilon \omega \cancel{E}$$

$$\therefore \sigma = \omega \epsilon$$

$$\omega = \frac{\sigma}{\epsilon} = \frac{100}{4 \times 8.85 \times 10^{-12}} = 2.82 \times 10^{12} \text{ rad/s}$$

Time Harmonic Fields

Field vary periodically or sinusoidally with time.

So any vector of EM wave has complex phasor & expressed by

$$\boxed{\bar{A}(t) = A_o e^{j(\omega t + \theta)}} = A_o \cos(\omega t + \theta) + j A_o \sin(\omega t + \theta)$$

لكتابة ال phasor نزيل $e^{j\omega t}$

ويبقى ال phasor $A_o e^{j\theta}$

Real part of $\bar{A}$ $= A_o \cos(\omega t + \theta)$

Imaginary " " $= A_o \sin(\omega t + \theta)$

instantaneous form

Ex(04) Express the following vectors in phasor form

(i) $E_y(z,t) = 100 \cos(10^8 t - 0.5 z + 30^\circ)$ V/m

(ii) $\bar{A}(x,t) = 10 \cos(10^8 t - 10 x + 60^\circ)\, a_z$

sol

step 1 write exponential eq^n as follows:

$$E_y(z,t) = Re\ 100\, e^{j(10^8 t - 0.5 z + 30)}$$

step 2 drop Re & suppress $e^{j 10^8 t}$ to obtain phasor

$$E_y(z) = 100\, e^{-j 0.5 z + j 30}$$

6

b) $\bar{A} = 10\cos(10^8 t - 10x + 60^\circ)$

$= Re\ 10 e^{j(10^8 t - 10x + 60)} \Rightarrow$

$$\boxed{\text{phasor} = 10 e^{-j10x + j60j}} = \bar{A}_s(x)$$

EX(05) express the following vector in instantaneous form

$$\bar{B}_s(x) = \left(\frac{20}{j}\right)\hat{a_x} + 10 e^{j\frac{2\pi x}{3}} \hat{a_y}$$

Sol/ $\bar{B}_s = \frac{20}{j}\hat{a_x} + 10 e^{j\frac{2\pi}{3}}\hat{a_y} = -20j\,\hat{a_x} + 10 e^{j\frac{2\pi}{3}}\hat{a_y}$

نضرب $e^{j\omega t}$ ×١ / فاخذ الجزء الحقيقي

$\bar{B}(x,y,t) = Re(B e^{j\omega t})$

$$a + jB = \sqrt{a^2+B^2}\, e^{j\tan^{-1}(\frac{B}{A})}$$

$\therefore$ ~~$\bar{B}_s(x,y,t) = Re(2$~~

$\therefore B_s = -20j\,\hat{a_x} + 10 e^{j\frac{2\pi}{3}}\hat{a_y}$

$B_s = 20 e^{-j\frac{\pi}{2}}\hat{a_x} + 10 e^{j\frac{2\pi}{3}}\hat{a_y}$

$B(x,y,t) = Re\left[20 e^{-j\frac{\pi}{2}}\hat{a_x} + 10 e^{j\frac{2\pi}{3}}\hat{a_y}\right] e^{j\omega t}$

$$\boxed{B(x,y,t) = 20\left[\cos(\omega - \tfrac{\pi}{2})\,\hat{a_x}\right] + 10\cos(\omega t + \tfrac{2\pi}{3})\,\hat{a_y}}$$

EX(06) for EM in free space with $\rho_v = J = \sigma = 0$

If $E(z,t) = 0.5\cos(2\pi \times 10^9 t - 20.9z)\,\hat{a_x}$ V/m

find ① $\vec{E}$ in phasor form ② H

Sol $\bar{E}(z,t)$ phasor $= Re\ 0.5 e^{j(2\pi\times10^9 t - 20.9z)}\hat{a_x}$

i) $$\boxed{\bar{E}_{zs}(z) = 0.5 e^{-j20.9z}\,\hat{a_x}}$$

(ii) $\because \nabla \times \bar{E} = -j\omega \bar{B} \Rightarrow \boxed{\nabla \times \bar{E} = -j\omega\mu H}$

$$\nabla \times \bar{E} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \begin{vmatrix} \hat{a_x} & \hat{a_y} & \hat{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0.5 e^{-j20.9z} & 0 & 0 \end{vmatrix} = 0\hat{a_x} - (0.5)(j20.9)e^{-j20.9z}\,\hat{a_y} + 0\hat{a_z}$$

$$\nabla \times \bar{E} = -0.5 \times j\, 20.9\, e^{j-20.9z}\, \hat{a}_y$$

$$j\omega\mu H = \nabla \times \bar{E} \quad \therefore \quad H = \frac{1}{j\omega\mu} \nabla \times \bar{E}$$

$$, H_x = H_z = 0 \quad \& \quad H_y = \frac{10.47}{\omega\mu} e^{-20.9z}$$

Ex(07) من 6 دارات